**Time limit:** 60 minutes.
**Instructions:** This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise.
**No calculators.**

1. Towa has a hand of three different red cards and three different black cards. How many ways can Towa pick a set of three cards from her hand that uses at least one card of each color?

2. Alice is counting up by fives, starting with the number 3. Meanwhile, Bob is counting down by fours, starting with the number 2021. How many numbers between 3 and 2021, inclusive, are counted by both Alice and Bob?

3. How many distinct sums can be made from adding together exactly 8 numbers that are chosen from the set $\{1, 4, 7, 10\}$, where each number in the set is chosen at least once? (For example, one possible sum is $1 + 1 + 1 + 4 + 7 + 7 + 10 + 10 = 41$.)

4. Derek and Julia are two of 64 players at a casual basketball tournament. The players split up into 8 teams of 8 players at random. Each team then randomly selects 2 captains among their players. What is the probability that both Derek and Julia are captains?

5. How many three-digit numbers $\underline{abc}$ have the property that when it is added to $\underline{cba}$, the number obtained by reversing its digits, the result is a palindrome? (Note that $\underline{cba}$ is not necessarily a three-digit number since before reversing, $c$ may be equal to 0.)

6. Compute the sum of all positive integers $n$ such that $n^n$ has 325 positive integer divisors. (For example, $4^4 = 256$ has 9 positive integer divisors: 1, 2, 4, 8, 16, 32, 64, 128, 256.)

7. For a given positive integer $n$, you may perform a series of steps. At each step, you may apply an operation: you may increase your number by one, or if your number is divisible by 2, you may divide your number by 2. Let $\ell(n)$ be the minimum number of operations needed to transform the number $n$ to 1 (for example, $\ell(1) = 0$ and $\ell(7) = 4$). How many positive integers $n$ are there such that $\ell(n) \leq 12$?

8. Consider the randomly generated base 10 real number $r = 0.\overline{p_0p_1p_2\ldots}$, where each $p_i$ is a digit from 0 to 9, inclusive, generated as follows: $p_0$ is generated uniformly at random from 0 to 9, inclusive, and for all $i \geq 0$, $p_{i+1}$ is generated uniformly at random from $p_i$ to 9, inclusive. Compute the expected value of $r$.

9. Let $p = 101$. The sum
$$\sum_{k=1}^{10} \frac{1}{\binom{p}{k}}$$
can be written as a fraction of the form $\frac{a}{p!}$, where $a$ is a positive integer. Compute $a \pmod{p}$.

10. Let $N$ be the number of ways to draw 22 straight edges between 10 labeled points, of which no three are collinear, such that no triangle with vertices among these 10 points is created, and there is at most one edge between any two labeled points. Compute $\frac{N}{9!}$.